Learning Symmetry: A Shape from Shading Approach[*†]

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Abstract

Given that an object or scene is symmetric, suitable geometric and/or photometric constraints can be derived which aid in machine understanding of its images. There are several examples of application of symmetric assumption [4][3][1][2] in Computer Vision literature but hardly any of them emphasize on evaluating the symmetry of the scene. Quite clearly, symmetric assumption when forced on an asymmetric scene may result in incorrect inferences. In this paper, we propose a theoretical formulation to verify the symmetry of a scene or object given just one image. The symmetric points in a scene will in general have different intensity values in an image due to asymmetric placement of illumination sources which makes the problem non-trivial. The Shape from Shading formulation we present, models the physical process of image formation, thereby making it possible to evaluate symmetry of an object given just one image taken under arbitrary illumination source.

Description

We focus only on bilateral symmetry of the imaged object or scene. As with the standard Shape from Shading (SfS) formulation, Lambertian reflectance is assumed. Here, each point is characterized by its albedo ρ and surface gradients $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. Without loss of generality, we assume that the scene is viewed from $(0, 0, -1)$ and the axis of symmetry lies in a plane parallel to the image plane. Therefore, the albedo ρ_- and surface gradients $\{p_-, q_-\}$ of the bilaterally symmetric point are characterized as follows

$$\rho_- = \rho \qquad \{p_-, q_-\} = \{-p, q\} \tag{1}$$

Under the assumption of orthographic projection, the perceived intensity of a surface point of the imaged object can be written as

$$I = L\rho \frac{1 - pl - qk}{\sqrt{p^2 + q^2 + 1}\sqrt{l^2 + k^2 + 1}} \tag{2}$$

where ρ is the surface albedo, $\frac{(p,q,1)}{\sqrt{p^2+q^2+1}}$ is the surface normal, L is the intensity of the light source and $\frac{(l,k,1)}{\sqrt{l^2+k^2+1}}$ is the illuminant direction. As done normally in SFS formulations, we assume that the image intensity I is normalized by the known light source intensity to eliminate L from the expression. Similarly, intensity I_- of the corresponding symmetric point can be written as follows

$$I_- = \rho \frac{1 + pl - qk}{\sqrt{p^2 + q^2 + 1}\sqrt{l^2 + k^2 + 1}} \tag{3}$$

[*] Category: Visual processing and pattern recognition
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Using (2) and (3), the albedo can be eliminated leading to the following linear constraint on the surface gradients

$$\frac{I_-}{I} = \frac{1 + pl - qk}{1 - pl - qk} \tag{4}$$

$$(I_- - I) - (I_- + I)pl - (I_- - I)qk = 0 \tag{5}$$

$$Slp + Dkq = D \tag{6}$$

where $S = I_- + I$ is the sum of the intensities of the symmetric points and $D = I_- - I$ is the difference of the two. The linear relation implies that the set of possible surface gradients $\{p, q\}$ lie on a straight line in the pq-space, parameterized by the perceived intensity and the lighting condition. It is worthwhile to note that the regular reflectance map (2) provides a quadratic constraint on the values surface gradients can take, given the pixel intensity, albedo and illumination conditions. Figure 1 shows the regular quadratic reflectance map and the corresponding linear constraints (6).

In general, the albedo value is not known. Therefore, the regular reflectance map can only restrict the unknown surface gradients $\{p, q\}$ to a family of conic sections, one conic section for each speculated value of the unknown albedo value. If we consider a pair of bilaterally symmetric points, their conic sections corresponding to the same speculated albedo value intersect on the line given by (6). The same is true for all speculated albedo values covering the whole family of conic sections. Therefore, if two points are bilaterally symmetric, the locus of these intersection points is a line given by (6). On the other hand, if the two points are not bilaterally symmetric, we get a relation of the form

$$\frac{I_-}{I} = \frac{\rho_-}{\rho} . \frac{1 - p_- l - q_- k}{1 - pl - qk} . \sqrt{\frac{p^2 + q^2 + 1}{p_-^2 + q_-^2 + 1}} \tag{7}$$

Quite clearly, this is different from the one in (6) and is not even linear. Therefore, two corresponding conic sections resulting from points which are not symmetric will not intersect on the line (6). Moreover, in such a case, the locus of intersection points of the conic sections we mentioned is not going to be a straight line. These facts can easily be used to evaluate the symmetry of the imaged scene or object.

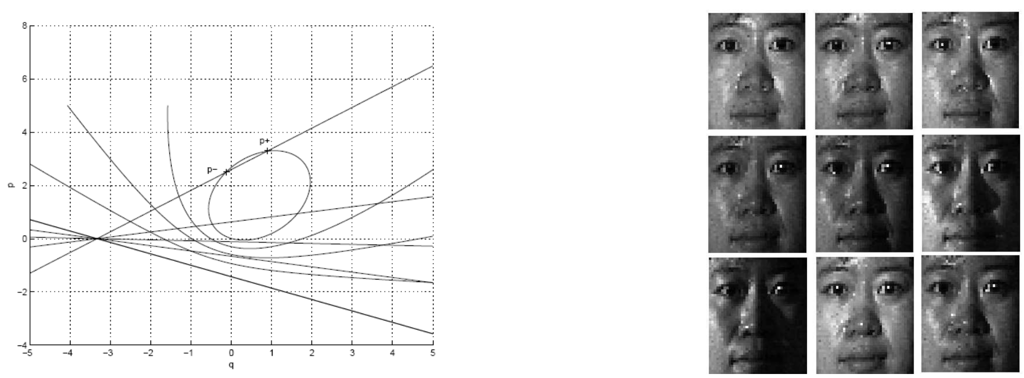

Figure 1. Left: Regular and symmetric reflectance maps. Right: A few images showing how even symmetric points can have very different intensity values making the problem of evaluating symmetry non-trivial.

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